Hurd, W.C.
1935

A study of individual differences and failures in algebra

School of Education
June 10, 1935
13457

ACCOPRESS BINDER
USE ACCO PAPER FASTENERS FOR BINDING

No. BF 250-P7

MADE BY ACCO PRODUCTS, INC.
LONG ISLAND CITY, N. Y., U. S. A.

Ed.
Thesis
Hurd
1935

Stored

BOSTON UNIVERSITY

SCHOOL OF EDUCATION

Thesis

A STUDY OF INDIVIDUAL DIFFERENCES
AND
FAILURES IN ALGEBRA

Submitted by

W. Chandler Hurd
(Ed. B., Keene Normal, 1932)

In partial fulfillment of requirements for the
degree of Master of Education
1935

Boston University
School of Education
Library

First reader - Guy M. Wilson, Professor of Education
Second reader - Roy O. Billett, Associate Professor of Education

CONTENTS

Purpose of the Study 1

Chapter I. Individual Differences 3

Chapter II. Algebra Taught in the High School 13

Chapter III. The Study 30

General Conclusions 52

Conclusions on Study 53

Appendix 54

PURPOSE OF THE STUDY

The quality of the teaching that is done in mathematics is not above reproach, but it is possible that there are other factors than teaching and the intelligence of the pupil which enter into the large number of failures. The purpose of this study is to discover, if possible, whether or not there are other elements entering into the large number of failures in algebra, and, if so, what in general these deficiencies are. If it is possible to discover that certain mistakes in the four fundamentals, factoring, or fractions are causing many of the failures it may be possible for teachers in the future to give special attention to these phases of algebra and by so doing overcome some of the failures. It is not expected to furnish conclusive data nor a panacea that will allay all ills, but if there are any set type of mistakes that the majority of a group have difficulty with, this discovery should aid in planning for provision against failure in the future.

Some pupils may be failing in material in algebra that has been found unnecessary. If this is true it would be good pedagogy for those who construct the curriculums to take heed to the suggestions of the National Committee on the Reorganization of Secondary Education and other leaders who are revising the curriculums. This study will include some of these recommendations.

Individual differences play an important role in education. A survey of some of the outstanding differences and some of the plans for overcoming the main differences in teaching classes which contain a large number of individualities may be of use to those concerned with this phase

Digitized by the Internet Archive
in 2017 with funding from
Boston Library Consortium Member Libraries

https://archive.org/details/studyofindividua00hurd

of teaching. Some of the plans should be of use in local situations.

All studies have certain limitations and this one is not an exception. The pupils under surveillance are all taken from one high school. The group was failing at the time that the study began. It is not to be inferred that the mistakes that this group made will be the ones that all groups would necessarily make, but it would be probable that the mistakes that the majority of a group of twenty-five pupils made would be characteristic of those that others might make.

The proposed material to be included in the algebra classes has its limitations. A group taking a scientific course may need different algebra from that needed by an academic group. However, this difference may be left to the discretion of the administrators of any said school.

Furthermore the study did not cover a long enough period of time. A study which would cover the entire year would be able to more justifiably offer suggestions concerning mistakes and failures as that study could follow the difficulties over a longer period of time thus allowing for a more accurate diagnosis.

Chapter I.

INDIVIDUAL DIFFERENCES

Individual differences play a large part in school failures. Whether it is the individual himself or the inability of the teacher to cope with differences within the class it is difficult and indiscreet to generalize or prophesy. A study of some of the factors that compose these differences and some suggested means for alleviating them may aid in better visualization of the reasons for failure.

General intelligence is one factor which makes for differences in individuals. There is much justified dissension on what constitutes intelligence. It is a moot question even in the minds of psychologists. The inconsistency of various beliefs is due to the fact that intelligence is of an abstract quality. It is evasive; no positive means for measuring it has been conceived as yet. It is probably safe to remark that the individual test is more positive and trustworthy than the group test, although it has many acknowledged shortcomings.

Arthur Gates[1] states that there is a large range of differences in intelligence. Generalizations made from data collected by Terman and others affords the opportunity to estimate that the distribution of intelligence in the population at large is about that given in the following table:

I. Q. below 70-------------1%
I. Q. 70-79-------------5%
I. Q. 80-89------------14%
I. Q. 90-99------------30%
I. Q. 100-109------------30%
I. Q. 110-119------------14%
I. Q. 120-129-------------5%
I. Q. over 130-------------1%

Aside from these facts one would realize in his general acquaintance with

---

[1] Gates, Arthur I., Psychology for Students of Education, p. 507.

groups of people, if he were the least observing, that people do differ to some degree in intelligence.

Professor Thorndike is of the opinion that a pupil whose I. Q. is below 110 "will be unable to understand the symbolism, generalizations, and proofs of algebra. He may pass the course, but he will not really have learned algebra. This would rule out more than half (56%) of the present first-year students."[1] To add a bit of verification, possibly, to this statement the following investigation may be cited:

> "A recent investigation made by the department of psychology at Dartmouth College showed that all students of high rank in mathematics had a high rating on general intelligence; the converse was not true, however."[2]

It is probable that a relatively high intelligence is a prerequisite to taking a mathematics course, whether it be of the elementary type or higher mathematics. Some, at least, would deem it a judicious move not to permit all those desiring to take a mathematics course in high school to take it. If a prognostic or ability test in algebra were given at the beginning of the year there would be less "pulling" of pupils through the course and less discontent on the part of those who are able to cope with the subject matter of the study.

The percentage may be less than that which Professor Thorndike cited at the present writing, but even assuming this to be true, the fact still remains that there are many pupils taking algebra, and subsequently failing it, who should have been guided away from the course. There is ample room for improvement in counseling ninth grade pupils in what courses to select. The ability to do a prescribed piece of work is a prerequisite to satisfaction in one's self and the subject. Interest plays a large part in developing a feeling of satisfaction in an individual. If it is possible,

---

[1] Thorndike, E. L., The Psychology of Algebra, p. 321.

[2] The Reorganization of Mathematics in Secondary Education, Footnote, p. 61.

through correct motivation, to develop an interest in the subject, satisfaction and a possible ability to pass will result.

Douglass[1] says, in substance, that there have been many new developments which give a rather graphic insight into the universality of individual differences. Standard tests and scales for the measurement of school achievement have done much to impress educators with the large part that individual differences do actually play in an educational system and to see how much variation there is between pupils of the same age and grade. Individual intelligence tests (of which the Stanford Revision of the Simon-Binet is probably the most accurate at the present time) have lent further insight into the vast differences between people.

The school survey movement, as developed by Hanus, Cubberly, Strayer, Judd, and others in the decade of 1910-1920, also contributed much to bringing the facts as to individual differences to the foreground.

There are other differences in pupils than intelligence. Achievement, attitude, perseverance, and home conditions all tend to vary a pupil's adaptiveness to school subjects. All of these difficulties cannot be overcome with en masse treatment. It is often necessary and essential to deal with pupils in small groups or individually. H. T. Manuel says, "Education can never be accomplished en masse, or on the average. It is a personal, individual process. We must be careful not to confuse instruments and methods of scientific research with the instruments and methods of the art of teaching."[2]

Different pupils need various amounts of time in which to master a given topic. An incident in the writer's experience may well be cited here. A pupil seemed to have a comparatively good knowledge of how to

---

[1] Douglass, H. R., Modern Method in High School Teaching, pp. 462-463.
[2] Cited by: Sutherland, A. A., "Factors Causing Maladjustment of School to Individuals," 24th Yearbook, National Society for the Study of Education, p. 3.

solve equations, but it soon became quite obvious that he had not had enough practice and drill on the application of solving for an unknown when a fraction was involved. When the following example confronted him he was at a loss as to how to deal with it: $\frac{x}{7}=5$. He solved it as if it were $7x=5$. After being shown the correct procedure, he was able to solve similar examples without any difficulty. This solitary case would go to show that with a little more aid at the time of teaching the pupil would have been able to continue on a par with the rest of the group.

> "Much time is wasted by setting drill as a stint for all to perform regardless of the amount of the ability which has already been developed."[1]

Different interests and emotional reactions within the individual make for differences that need to be coped with in order to meet with the most desirable effects. Fears are a persistent source of difference in pupils. Fear of some portion of work will render it impossible for the pupil to cope with the topic. Fear of ridicule at the board or before the class will force many to fail rather than to attempt the unsavory task of speaking. As forms of fear are inherent it is desirable that there be some provisions made so that the pupil can either overcome this fear or do his work without the constant worry caused by fear.

The "will to learn" varies. Home influence plays a large part. It is not meant to infer that this should be used as an excuse for the teacher not to attempt to overcome it, however. Too often home conditions have to take the brunt of blame for deficiencies that should be coped with by the teacher. A case may well be cited of a freshman who was failing but had the ability to do the work if only he had the desire to study. His attitude was "I'm going to fail anyway, so why should I try?". This attitude persisted for a number of weeks when suddenly there was a noticeable change in him.

---

[1] Sutherland, A. A., op. cit., p. 4.

He began to take an interest in his work and his accomplishments began to show color. He continually inquired if he still had a chance to pass and when he was assured that he had if he continued to do good work he began to put forth real energy. Tests began to show one hundred per cent accuracy. It is imperative that a teacher promote a feeling of hope in the pupil if the best results are desired. Perhaps a different attitude toward marks would afford the opportunity to aid the pupil more. The experiment of abolishing all marks is worthy of note.[1] The inference is not that a change as radical as this should be attempted, but rather that a different type of emphasis be placed on the mark of a failing pupil.

Many studies have been made in an attempt to discover the influence of class size upon pupil accomplishment. The larger the class the more complex will be the techniques employed to cope with individual differences of the group. This will necessitate more adequately trained teachers who can see and allow for the differences within the group. Certain teachers have an inherent capacity to discover, or rediscover talent and ability that is lying dormant in their pupils. Those who do not have this talent and aspire to be clever teachers may attain it through the many professional courses that are now being offered in universities and colleges.

> "By reading and study, self-directed, it may be that, in the course of time, the majority of teachers may prepare themselves to discover how far the various abilities of a child have developed, to measure the amounts of growth he makes under direction and ultimately to set up standards for each child, regardless of the standard of the room in which he sits."[2]

However, there should be some other incentive for doing this supplementary work than the satisfaction that it affords the teacher. In California a new type of certificate is being tried as an incentive to such study. In the teaching profession there is a need for further study on the

---

[1] Superintendent Lund.

[2] Jensen, M. D. and D. W., "The Influence of Class Size Upon Pupil Accomplishment in High School Algebra," Journal of Educational Research, XXI (Feb. 1930), p. 136.

part of those in the profession to gain more advanced theories and practices. This theory is being followed in other professions to a much greater degree than in the teaching field. There is likewise a decided need for a more stimulating remuneration.

In conjunction with the necessity for better teaching in large classes further studies have been made concerning the influence of the size of the class upon pupil accomplishment. It has been a problem that has been the cause for many experiments. It has caused much conjecturing and debating on the part of teachers and educators. Although experiments have been tried there have been too many elements, irrelevant and otherwise, entering into these experiments to allow for any final judgments as to the size of the class on pupil accomplishments to be passed. One extensive investigation related to high schools has been reported.

> "The North Central Association experiment, uncritically judged, might lead to the conclusion that, on the basis of school marks, in English, pupils progress best in classes of twenty or under; in modern language, from twenty to thirty; and in social science and mathematics, over thirty. In a similar study, Stevenson found no consistent tendencies in Chicago high schools.
>
> "Criticisms which may legitimately be made of the majority of reported studies include: failure to control such factors as sex, possible diurnal changes, presence of repeaters in experimental groups, size of room,----and initial accomplishment. Some of the tests used have low reliabilities and inadequate testing range for the group with which they were used; no attempt has been made to determine statistically the significance of reported differences, etc."[1]

This experiment would seem to refute the argument that it is difficult to teach algebra to a large class. However, the abilities of the pupils in a class and the capabilities of the teacher would make a big difference in the probable success of any one teaching procedure. It would be well to bear the above experiment in mind when selecting a group to compose a class.

---

[1] Ibid., p. 137.

Various methods of dealing with individual differences have been developed. Some facts that will be of helpful import may be gleaned from a study of a few of these methods. At present there have been two main teaching methods developed which deal with this problem of individual differences in learning the homogeneous grouping of classes on the basis of ability and the individual instruction method. The earlier attempts at homogeneous grouping were known as the Batavia plan and the Pueblo plan in which groups of students within a class were treated as individual units and were taught and promoted as groups. The advent of intelligence tests marked the period for the beginning of segregation into class groups on the basis of ability. The individual instruction method from printed material is more recent in its development and is now known as the Dalton plan, the contract plan, the Winnetka plan, the McDade plan, etc. All of these are characterized by individual progress and promotion.

It would not be inappropriate at this time to consider some of the predominating plans which tend to deal with individual differences and instruction. Some of the plans will deal directly with mathematics, others can be applied to mathematics classes with appropriate reorganization and proper application. Although P. M. Symonds[1] says, "As to 'ability grouping', not a single reliable experiment has been reported that warrants wholehearted imitation", there is the probability that a composition of more than one plan or an alteration of one may make an applicable plan for class procedure.

J. T. Johnson[2] describes a plan which has been tried and found successful at the Chicago Normal College during the past six years. He has dubbed it the "differentiated subject-matter plan."

---

[1] Symonds, P. M., Measurement in Secondary Education, p. 478.

[2] Johnson, J. T., "Adapting Instructional Material to Individual Differences in Learning," The Mathematics Teacher, XXVI, (April, 1933), pp. 193-199.

Various methods for the use of such a plan are expounded. Differentiated subject-matter is important. Courses have been surging like the tide, first toward the retention of high standards, and subsequently many failures, then receding toward low standards and fewer failures. It would seem that there has been too much indulgence in the lowering of standards in the recent past; the writing of easy courses in mathematics with the excuse of attempting to provide the many poorly prepared with a course which they can pass. This excuse is not a legitimate one as there are many pupils who desire a more complete knowledge of mathematics. This fact would justify a differentiated subject-matter course.

> "This method aims at keeping the essential benefits of the individual system and at the same time not destroy the class organization. It is felt that there are certain advantages accruing from the group work that should be retained besides saving time to the teacher as a new topic is presented."[1]

The following conclusions were drawn dealing with observed advantages of the method:

> "1. The feeling of success is the best promoter of interest. The student likes to do that in which he succeeds. As enough time is given the slow pupil he grows in self-confidence.
>
> 2. As units are so prepared that what is a prerequisite in one unit is mastered in a foregoing unit students begin each unit with the proper preparation.
>
> 3. The problem of discipline is reduced to a minimum, for each student is busy with work on his own level at every moment. The brighter students are challenged with work that requires their best efforts.
>
> 4. It reduces careless errors because each student is held to one hundred per cent mastery on every test before he can go on earning extra credit.
>
> 5. It induces the best attention during the presentation. This is due to the eagerness of all not to miss anything in order to get one hundred per cent on the first test.
>
> 6. It reduces failures in that the slower students are helped in corrective work and given more than one test. It provides as many as four different teachings on any one different point in any one unit.

---

[1] Johnson, J. T., op. cit., p. 198.

7. It has the advantage of the individual instruction methods without breaking up the organization of the class. It thus reduces the load and worry on the part of the teacher.

8. It is thoroughly democratic in that it gives everyone the same chance in every unit."[1]

M. A. Dalman[2] instigated a plan and tried an experiment in conjunction with the plan for the diminishing of failures in an algebra class in an Indianapolis high school. The large number of failures which prevailed caused this method to be conceived and tried. He seemed to be of the opinion that it had some good merits and was appreciably successful. The plan provides for individual differences. Each pupil delves further and further into the material although he continues to use the same principles and rules as the rest of the class.

The problem of individual differences was considered in the study in connection with failing pupils. The number of pupils in the groups was small enough so that the problem could be noticed more carefully than as though the groups had been larger. The amount of time devoted to the individual in the class is relatively small. Many of the errors and difficulties of the pupils were noted to be based upon the fact that the class is taught en masse with little or no consideration given to the individual pupil. This criticism seems justifiable in the light of the fact that when difficulties were noted and measures taken to correct them that pupils would grasp material when given individual attention whereas phases of the material remained hazy in the pupil's mind when all facts were endeavored to be made clear to the group as a whole. The differences in intelligence and personal traits were noted in each individual; the latter being discussed fully in the last chapter. This procedure allowed a relatively efficient method of considering the factors of individual differences as applied to

---

[1] Johnson, J. T., op. cit., p. 199.
[2] Dalman, M. A., "Hurdles, a Series of Calibrated Objective Tests in First Year Algebra," Journal of Educational Research, I, (Jan., 1920), pp. 47-62.

the study of algebra.

In coping with the problem of individual differences the teacher will find it necessary to determine what techniques and procedures will be best adapted to the group of pupils with which he is working. Probably the three most discussed plans are the Morrison, Dalton, and Winnetka. They are all adapted in various ways to the problem of individual differences, but have not been discussed here as they are well known to the average person interested in education. Often times it is found desirable to use a combination of more than one plan or only portions of a plan in order to have the class procedures agree with the ideas of the administration authorities. The plan or plans used depends on the teacher and what he knows will be the most desirable procedure to use from all viewpoints.

Chapter II.

ALGEBRA TAUGHT IN THE HIGH SCHOOL

In most schools in the United States algebra is studied before geometry. Algebra is sometimes introduced informally in the seventh or eighth grade, but the ninth grade sees it formally taught for the first time. It is generally supposed that at this time the pupils have reached a mental maturity which is capable of being adjusted to the reasoning required in algebra.

Endeavoring to overcome some of the inefficiencies of mathematical teaching some reformers wish to place geometry before algebra, although most of them advocate a simultaneous teaching of the subjects. Arthur Schultze, former lecturer at New York University on the Teaching of Mathematics in Secondary Schools, seems dubious about the desirability of merging the two subjects, although he does believe in introducing similar illustrations into each. This plan would not probably produce any radical improvements in the mastery of either subject as the reformers desire to believe.[1]

When considering the problem of individual differences and failures in algebra it is important to know what should be taught. Many subjects in the modern curriculum have been shorn of superfluous trimmings. Leaders in education have been endeavoring to promote the practice of having curriculums contain more of that which has practical value and less of the time-worn, impractical subject-matter which was supposedly the parent of a fertile mind. Much of the old theory of "formal discipline" has been cast aside.

---

[1] Schultze, Arthur, The Teaching of Mathematics in Secondary Schools.

That which should be taught in an algebra curriculum and that which should be pruned has provided educators with a moot question for years. It would be both impossible and impractical to attempt to teach all phases of algebra which will be used at some unforeseen date in physics, chemistry, mechanics, et cetera, for the course would be so crowded that no high school pupil could wade through it. There has been a tendency in all traditional subjects to emphasize the material of practical value. This is likewise true in algebra.

Arther Schultze gives the tendencies of these proposed changes as follows:

"1. Reduction of all information to a minimum: elimination of all superfluous, abstract, and merely technical matter.

2. Emphasis on all algebraic topics that require original thinking.

3. Emphasis on all topics that are frequently applied in geometry, physics, engineering, commercial problems, etc."[1]

It is imperative that the four fundamental processes of arithmetic be mastered as well as some of the other fundamentals of algebra which will be used as a basis for further study and performance in the subject. Purely formal work and much of the theory may well be reduced in the study of algebra. The former was included at first in the course of study for two reasons: first, it was taught in order to prepare pupils for formal examinations for college and, as the entrance requirements for colleges have undergone radical changes, this reason is much depreciated; secondly, it was thought to be of disciplinary value. This point has withered under the scathing attack of modern educationalists, philosophers, and progressive teachers. Theory of algebra, at least to those in elementary classes, is beyond the ken of the pupil and therefore not particularly desirable to

---

[1] Schultze, Arthur, op. cit., p. 292.

teach. The protagonists of the teaching of the more difficult phases of algebra to the pupils put forth the argument that in order to continue further study of mathematics theory must be taught. These do not take into consideration the element of individual differences and the fact that a relatively small percentage of the group will continue school after graduation and therefore have comparatively little use for the theory, except as a possible lever for self aggrandizement.

It is appropriate at this time to again refer to individual differences of pupils. It is possible to satisfy both groups through differentiation of pupil assignment and class procedure. Application of Dalman's Hurdles[1] or other procedures which tend to provide for individual differences would allow for the interested and the intellectually capable pupil to proceed rapidly and delve into algebra intensively as well as extensively thus allowing him to have a strong background.

The best evidence of the present trend or trends along the lines of "what to teach" is furnished by a brief survey of the contents of the recent texts prepared for the junior high schools. Such a survey reveals substantial agreement on the questions of content for the ninth year. There is evidence that the indictment contained in the memorandum addressed to the General Education Board by a committee appointed by the Mathematics Association of America no longer holds for a large number of progressive schools. Formal algebra of the type referred to in this memorandum is passing from our schools. This, however, does not mean that there is not ample room for more progress in most schools. The gist of the memorandum is expressed in the following passage:[2]

---

[1] Dalman, M. A., "Hurdles, A Series of Calibrated Objective Tests in First Year Algebra," Journal of Educational Research, I, (Jan., 1920), pp. 47-62.

[2] Schorling, Raleigh, A Tentative List of Objectives in the Teaching of Junior High School Mathematics with Investigations for the Determining of Their Validity, p. 2.

"The situation that needs to be met may best be illustrated by the case of algebra. Our elementary algebra is, in theory and symbolism, substantially what it was in the seventeenth century. The present standards of drill work, largely on non-essentials, were set up about fifty years ago. The few lines of application of algebra to nominally practical questions found in our college entrance examinations are mere variants of problems that are centuries old, and that often represent remotely any real conditions of today. A considerable number of teachers, in both the secondary schools and colleges, believe that the amount of time spent by pupils on abstract work in difficult problems in division, factoring, fractions, simultaneous equations, radicals and the like is excessive; that such work leads to nothing important in the science, and adds but little to facility in the manipulation of algebraic forms.

"It is urged by many friendly critics that instead of giving the students a good all-round idea of what mathematics means and its general range of application our present secondary schools are too abstract, often uninteresting, except to the mathematically inclined, and not as valuable as they might be as an aid to college in general, or to the life work for those who enter at once into their careers."

Much of the present mathematics in high school cannot function properly even in the hands of skilled teachers. It has not been revised sufficiently to allow for the immature minds of the adolescent pupil to cope with that which is demanded of him.

The adolescent period, into which much of elementary algebra is placed, is the "worst period between the shedding of the milk teeth and the grave for anything like drill",[1] unless properly motivated.[2] This agrees in substance with the aforesaid contention that algebra, as taught and outlined in courses of study prevalent at present, does not represent enough of the practical and too much of the organized drill material which tends to undermine the constitution of the adolescent.

However, he may consider the subject with enthusiasm if it has the tang of practicality to it. Much of the difficulty in the curriculum is not in the abstractness of the concepts but rather, "(1) in the extreme difficulty of finding an opportunity for them (the concepts) to function, and

---

[1] Morrison, H. C.,"The Adaptation of Instruction to the Needs, Interests, and Capacities of Students,"Thirteenth Yearbook, National Society for the Study of Education, p. 12.
[2] Gates, A. L., Psychology for Students of Education.

(2) in the fact that algebra as taught is almost entirely an organizing and drill subject."[1]

Technical high schools should teach those processes which are needed in the industry or vocation for which it is training, followed by drill to the efficient functioning within a certain range.[2]

In a discussion with a headmaster of a high school the writer found that the preceding paragraph was, in substance, one of the criticisms that he found in the teaching of present mathematics. Teachers in physics and chemistry discovered that it was necessary to reteach some phases of the formula in order that the pupil be able to correctly do problems. It would seem that the problems as taught in algebra were not reaching the pupil as they should. Oftentimes when something which is of practical significance is taught from the theoretical viewpoint the pupil is unable to grasp how to apply it when the opportunity arises. This criticism would seem to lend weight to the idea that those processes which will be used later should be drilled to the point of efficient functioning. The problem, of course, arises as to what constitutes "the necessary". It must of necessity be somewhat of a local problem with each administration as there may be special requirements. And teachers of physics must teach algebra as needed in physics.

In teaching a subject which has possibilities of its fundamentals being applied in other fields there should be a building-up of adaptive controls of conduct -- that knowledge which can be applied intelligently to other phases of the work. If this is not developed the subject-matter will tend to be shallow and inconsequential. A variance from the theoretical to the practical will be one means of developing a more valuable type of

---

[1] Morrison, H. C., op. cit., p. 13.
[2] Ibid., p. 17.

adaptive controls in the pupil.

Mathematics like language is essentially a "tool subject" and should be taught and treated as such. By teaching that which is applicable in a subject a better appreciation and understanding of its lasting practical values may be inculcated. That which connotes applicability tends to create interest, and interest in a subject will, in all probability, develop in the student a greater desire to acquire the necessary knowledge and skill dealing with the subject-matter.

Morrison[1] in his study drew the following conclusions:

> 1. .....Mathematics must be revised as to organization and content, and adapted to the known nature of the adolescent and to the social purpose of the high school as that purpose is increasingly revealed by modern conditions.
>
> 2. Mathematics must be treated primarily as a language, the purpose of which is the interpretation of the various sciences.
>
> 3. Courses in mathematics must be arranged at such points in the curriculum as will give immediate opportunity for functioning.
>
> 4. The several integral parts of the program such as household arts, etc., must each have its own specially organized mathematics; and the mathematics of each curriculum should probably be in charge of the specialists of that curriculum rather than in the hands of a separate mathematics faculty.

Edwin S. Lide[2] has made a study of prevailing practices of a selected group of schools in the United States. A topical analysis was made of twenty-eight courses in algebra and twenty-four courses in general mathematics, as in grade nine. These data showing the percentage of schools offering the various topics, the order in which the topics are presented, and the average number of weeks allotted to each, are shown in the following table.

---

[1] Morrison, H. C., op. cit., pp. 30-31.

[2] Lide, Edwin S., "Instruction in Mathematics," National Survey of Secondary Education Bulletin, 1932, No. 17, Monograph No. 23, p. 34, Table 10.

Table I.

TOPICS INCLUDED IN NINTH-GRADE ALGEBRA AND GENERAL MATHEMATICS IN THIRTY PER CENT OR MORE OF ALL OUTLINES, THE ORDER IN WHICH PRESENTED TO THE CLASS, AND THE NUMBER OF WEEKS ALLOTTED TO EACH.

	Algebra			General Mathematics		
Topic	Percent of Outlines Showing (28)	Average of the Order in which Presented (28)	Average Number of weeks of time Allotted (14)	Percent of Outlines Showing (24)	Average of the Order in which Presented (24)	Average Number of weeks of time Allotted (14)
1	2	3	4	5	6	7
Fundamental operations of algebra.....	89	1	4.9	54	5	5.0
Factoring...........	86	4	4.4	67	8	4.5
Fractions...........	86	6	5.5	67	7	4.5
Exponents & radicals.	86	9	3.4	42	13	3.0
Equations, its use...	79	2	3.8	54	3	3.3
Simple quadratics....	71	10	2.7	50	19	2.5
Graphs and graphical representation.......	71	7	1.9	50	4	1.0
Positive and negative numbers.........	68	3	1.7	71	2	---
Linear equations.....	64	8	2.8	42	9	3.0
Formula, meaning and use..................	57	5	1.0	79	1	2.3
Application of ratio and proportion.......	39	11	2.0	58	10	2.0
Equations in two unknowns............	30	12	3.0	38	21	3.0
Trigonometry.........	--	--	---	54	38	5.0

Note:-The number in ( ) indicates the number of outlines represented.

Comparisons of the content of courses in algebra and in general mathematics have been made through studies in which textbooks have been analyzed. McCormick made a study of twelve texts in algebra published since 1923 and compared them with eight texts in general mathematics for the ninth grade. The following table presents the mean percentages which he computed for the different branches.

Table II.

MEAN PERCENTAGES BY WHICH DIFFERENT BRANCHES ARE INCLUDED IN CERTAIN TEXTBOOKS IN ALGEBRA AND GENERAL MATHEMATICS, GRADE NINE.[1]

Branch	Mean Percentages Algebra	 General Mathematics
Arithmetic........................	5.0	5.3
Statistics, computational.........	.3	1.1
Statistics, graph.................	3.9	3.6
Algebra...........................	76.0	58.2
Algebra, graph....................	4.6	5.3
Geometry, intuitive...............	6.2	22.5
Geometry, demonstrative...........	-0-	1.8
Trigonometry......................	3.9	2.1
Total	99.9	99.9

The summary indicates that considerably more attention is devoted to algebra and a little more to trigonometry in textbooks in algebra, but that more attention is given to arithmetic, statistics, and geometry in textbooks in general mathematics.

Another study in which changes in the content of elementary algebra from 1818 to 1928 are considered is that of Chateauneuf,[2] in which

---

[1] Op. cit., Monograph No. 23, p. 36, Table 11.

[2] Chateauneuf, A. O., Changes in the Content of Elementary Algebra Since the Beginning of the High School Movement as Revealed by the Textbooks of the Period, pp. 140-142.

257 texts were analyzed. Her data are presented by ten-year periods and show that the most significant changes between the periods 1910-1920 and 1920-1928 are an increase in the per cent of space devoted to general equations and formulas and a decrease in the space devoted to factoring, factors, and multiples. In comparing changes of content materials from 1890 to 1928 with the recommendations of the National Committee, she shows that they are favorable along the following lines:

1. An increase in both number and percentage of verbal exercises.
2. A decrease in the per cent of tool exercises (although there was an increase in the mean amount.)
3. A decrease in the emphasis on complicated algebraic technique.
4. Limitation of drill to certain processes deemed necessary by the Committee.[1]

These general tendencies in the revision of textbooks go to show that in many current courses there are still some traditional materials that could well be omitted. When the point where texts begin to revise the material is reached, it would seem that existing age-worn courses should be revised to conform with these changes.

The National Committee on Mathematical Requirements[2] believes that the courses in mathematics for the three years, seven, eight and nine, should be planned as a unit "with the purpose of giving each pupil the most valuable mathematical training he is capable of receiving in those years, with little reference to courses which he may or may not take in succeeding years." College entrance requirements should not be waved before the pupil as a goal which he must achieve. In following courses of mathematics the desired essentials for passing the examinations may be mastered. This resolution seems very logical and in keeping with later philosophies which

---

[1] Chateauneuf, A. O., Ibid., p. 142.

[2] Reorganization of Mathematics in Secondary Education, (Part I), p. 19f.

have been propounded. The junior high school (for which the National Committee passed a resolution sanctioning it very highly) is both an experimental and orienting ground. By demanding too strict requirements oftentimes the germ of discovery is severely incapacitated if not killed. Most certainly this fact would never be upheld as an objective and yet it seems to be a far too common result.

The National Committee in studying the desired reorganization necessary in mathematics delimited the requirements of courses. In doing this it was necessary to have the proposed course coincide with College Entrance Requirements and, therefore, the argument that pupils taking the abbreviated course will not be able to pass the Entrance Examinations can be declared nil. The National Committee determined that the following should be the main essentials to be mastered in algebra:[1]

1. The formula -- its construction, meaning, and use (a) as a concise language; (b) as a shorthand rule for computation; (c) as a general solution; (d) as an expression of the dependence of one variable upon another.

The pupil will already have met the formula in connection with intuitive geometry. The work should now include translation from English into algebraic language, and vice versa, and special care should be taken to make sure that the new language is understood and used intelligently. The nature of the dependence of one variable in a formula upon another should be examined and analyzed with a view to seeing "how the formula works."

2. Graphs and graphic representation in general -- their construction and interpretation in (a) representing facts (statistical, etc.); (b) representing dependence; (c) solving problems.

---

[1] Reorganization of Mathematics in Secondary Education, (Part I), p. 32f.

After the necessary technique has been adequately presented graphic representation should not be considered as a separate topic but should be used throughout, whenever helpful, as an illustrative and interpretative instrument.

(This seems to be an important removal from the characteristic conventional means of teaching the subject. It affords more practical application of learned techniques which is always a desirable feature in a child's accumulation of knowledge. Also the correlation of various adaptive controls of conduct provides necessary opportunity for the pupil to apply knowledge formerly acquired.)

3. Positive and negative numbers -- their meaning and use: (a) as expressing both magnitude and one of two opposite directions or senses; (b) their graphic representation; (c) the fundamental operations applied to them.

4. The equation -- its use in solving problems:

(a) Linear equations in one unknown -- their solution and applications.

(b) Simple cases of quadratic equations when arising in connection with formulas and problems.

(c) Equations in two unknowns, with numerous concrete illustrations.

(d) Various simple applications of ratio and proportion in cases in which they are generally used in problems of similarity and in other problems of ordinary life. In view of the usefulness of the ideas and training involved, this subject may also properly include simple cases of variations.

5. Algebraic technique:

(a) The fundamental operations. Their connections with the rules of arithmetic should be clearly brought out and made to illuminate numerical processes. Drill in these operations should be limited. In particular, "nests" of parentheses should be avoided, and multiplication and division should not involve much beyond monomial and binomial multipliers, divisors and quotients.

(b) Factoring. The only cases that need to be considered are (i) common factors of the terms of a polynomial; (ii) the difference of two squares; (iii) trinomials of the second degree that can be easily factored by trial.

(c) Fractions. Here again the intimate connection with the corresponding processes of arithmetic should be made clear and should serve to illuminate such processes. The four fundamental operations with fractions should be considered only in connection with simple cases and should be applied constantly throughout the course so as to gain the necessary accuracy and facility.

(d) Exponents and radicals. The work done on exponents and radicals should be confined to the simplest material required for the treatment of formulas. The laws for positive integral exponents should be included. The consideration of radicals should be confined to transformations of the following types:

$\sqrt{a^2b}=a\sqrt{b}$, $\sqrt{a/b}=\frac{1}{b}\sqrt{ab}$ and $\sqrt{a/b}=\sqrt{a}/\sqrt{b}$, and to the numerical evaluation of simple expressions involving the radical sign. A process for finding the square root of a number should be included, but not for finding the square root of a polynomial.

(e) Stress should be laid upon the need for checking solutions.

The following topics the Committee suggested should be omitted or postponed until a later time, perhaps, except in the case of a few of the pupils, until the course in advanced algebra is given:

Highest common factor and lowest common multiple, except the simplest cases involved in the addition of simple fractions.

The theorems on proportion relating to alternation, inversion, composition, and division.

Literal equation, except such as appear in common formulas, including the derivation of formulas and the geometric relations, or to show how needless computation may be avoided.

Radicals, except as indicated previously.

Square root of polynomials.

Cube root.

Theory of exponents.

Simultaneous equations in more than two unknowns.

The binomial theorem.

Imaginary and complex numbers.

Radical equations except such as arise in dealing with elementary formulas.

The problem is perhaps the one phase of both arithmetic and algebra that is open to the most criticism by all who desire to have the most important elements of the subject taught. Problems may present to the pupil the best type of life situation if they are of the correct type. Those which are beyond the understanding of the pupil are more detrimental than helpful because they do not provide him with a problem in the true sense of

the word. The problem should be both "practical" and "real" to the pupil. If it is a problem that would be thought-provoking to a lumber-jack it will prove of little real value to the city dweller because he has no conception whatever as to the probable number of board feet in a specified log. The problem, instead, should take something that he will be somewhat acquainted with and have some conception as to the probable correct answer. If a survey were to be made of the prevailing textbook problems as to their practicality to the student the conclusions would probably be that they offered him little or no "problem". "The educational utility of problems is not to be measured by their commercial or scientific value, but by their degree of reality to the pupils.... They must exemplify those leading ideas which it is desired to impart, and they must do so through media which are real to those who are under instruction. The reality is found in the students, the utility in their acquisition of principles."[1]

> "There should be, moreover, a conscious effort through the selection of problems to correlate the work in mathematics with the other courses of the curriculum, especially in connection with courses in science. The introduction of courses in 'general science' increases the opportunity in this direction."[2]

The writer has not made a study of problems in algebra but the above quotations would indicate that there is room for a study of this phase of algebra.

The new philosophy of education of which Harry C. Barber speaks in his article, "Reorganized Mathematics in the Junior High School,"[3] gives added strength to the premise that correct, stimulating material should be given the pupil to study.

> "The new philosophy may best be characterized by the words 'common sense'. (No further attempt to define it can be made here). In education it says that we should work with nature rather than against her; that we

---

[1] Carson, Mathematical Education, pp. 42-50, Cited in Reorganization of Mathematics in Secondary Education, p. 39.

[2] Ibid., p. 39.

[3] Barber, Harry C., Second Yearbook, National Council of Teachers of Mathematics, p. 125.

should put the child and his interests at the center; that the best mental training comes from the pursuit of subjects interesting and useful to the child, under the guidance of good teaching."[1]

The article further states that the true value of elementary algebra is in the attitudes and intellectual experiences that may be derived from the study if it is taught correctly. It is invariably true that the type of instruction that a pupil receives makes a great deal of difference in the values which he will receive. This article, which was written four years after the preceding quotations from the monographs on "The Reorganization of Mathematics in Secondary Education", agrees in substance with the requirements that have been quoted. Added emphasis has been placed on formulas and equations which, it would seem, should constitute the bulk of the work in the ninth year.

"The complexity of the task of teaching signed numbers is now well recognized. The new psychology clarifies the situation. It tells us to introduce signed numbersas they are needed."[2]

The probable conclusion that can be drawn from this statement is that there is too much of the using of the text as an end in itself and not as a means to an end. The text is often relied on to the extent that it becomes a crutch that soon tends to develop a chronic dependence which may, in time, prove rather fatal to the pupil.

It would not be amiss to bring in at this time a paragraph on the importance of eighth grade algebra. If a carefully conducted experiment were to be performed with two unselected groups, one having algebra in the eighth grade and the other being introduced to it for the first time in the ninth year, it would prove an important contribution. Something of this nature was tried in the Horace Mann School for Boys in New York City. Algebra, up to quadratics, was taught in the eighth grade and in the ninth

---

[1] Second Yearbook, National Conncil of Teachers of Mathematics, p. 135.
[2] Ibid., p. 139.

year the subject was completed. It seemed true, however, that the pupils were unable to retain the more difficult portions of simultaneous equations which they had in the last few months of the year. Ralph Beatley in his article, "Effect of Varying the Amounts of Arithmetic, Geometry, and Algebra Studied in the Junior High School", points out the fact that the pupils in the ninth year seemed to grasp better the significance of algebra after having had a preparatory course in the preceding year.[1]

> "The change in attitude which these pupils show toward their later work is highly significant, however. Whereas formerly the algebra of the ninth year was a bugbear and a source of low marks and much discouragement, it is rare indeed to hear the pupils even mention it now. They accept it as a matter of course and get good grades in it."[2]

He further states that a whole year is too much and that there should be considerable arithmetic in this year. This is in keeping with the Program of Studies for the State of New Hampshire from which the following quotation is taken:

> "Instruction in mathematics in this grade should cause the fundamental conceptions of algebra to become a part of the pupils' working mathematical equipment. Rote instruction which leaves the pupil with merely a set of verbal memories is to be avoided. The pedagogy is the same as that for the arithmetic of the first and second grades."[3]

It would seem that attacking algebra, which is a new and entirely different phase of mathematics to the average eighth grader, with the same form of pedagogy with which arithmetic is attacked in the elementary grades would be a good policy.

College entrance requirements have done more harm than good insofar as they have required of the entering student a knowledge of many facts that are at present considered unnecessary by many educators. They will continue to exert undue pressure on secondary teaching as long as pupils go from high schools to college and are required to take the examinations.

---

[1] Second Yearbook, National Council of Teachers of Mathematics, pp. 147-153.
[2] Ibid., p. 148.
[3] State Board of Education, 1924, p. 10.

Until these requirements are changed so as to compare more favorably with existing philosophies of education they will prove to be more of a detriment than an aid. It would seem that it would be possible to have cooperation between the colleges and the secondary schools. It is probably in the offing and the sooner that it is realized the better it will be for the high school graduates of future years. However, a study of the existing requirements shows that there is much negligence on the part of present high school courses in not taking advantage of some of the changes that have already taken place. Many current courses still have in their outlines that which has been pruned by the entrance requirements.

In conclusion it may be stated that the question of what to teach in algebra has been given much serious consideration. The superfluous, non-essential material is being replaced by material containing values that are of use in the future work of pupils. The much criticized "formal discipline" that algebra was intended to provide is not being sanctioned by educators. The technique of teaching the subject-matter of algebra shows decided progress.

Chapter III.

THE STUDY

The purpose of this study is to determine, if possible, some of the fundamental difficulties in elementary algebra. A group of pupils, numbering twenty-five, were chosen from the failing pupils in four different classes of elementary algebra. The I. Q.'s of these pupils ranged from 72 to 106 according to group intelligence tests. Those having I. Q.'s below 87 were disregarded in the study because it was felt that those below that level would affect the results because of their deficient intelligence. The I. Q. of 87 was arbitrarily chosen so as to have a fairly representative group and at the same time a reasonably large group from which to collect data. The total result, after pruning, was seventeen pupils.

Tests from the Hotz Algebra Scales, Form A, were administered with the results which are recorded on the table which appears on the following page. The type of test is given and the score which appears under each in the column is the total number of examples which were completed correctly according to the Key in the Manual. Although the scores may seem small it is because much of the material is new to the group and they had not, at the time, been initiated into this phase of algebra. The groups, barring one individual who was a repeater, had had the subject for only six months.

Table I.

THE NUMBER OF EXAMPLES DONE CORRECTLY PER PUPIL
IN EACH TEST OF HOTZ ALGEBRA SCALES, FORM A.

Pupil	Addition and Subtraction	Multiplication and Division	Problem	Equation and Formula
1	1	2	1	0
2	7	4	3	1
3	0	2	1	0
4	2	3	2	0
5	1	2	2	0
6	2	3	3	1
7	4	5	4	1
8	0	1	2	0
9	1	2	1	0
10	2	4	1	0
11	3	4	3	0
12	0	2	1	0
13	2	3	3	1
14	2	4	1	0
15	2	5	1	0
16	2	3	1	1
17	2	1	1	0

This table shows that in the six months of study that the groups have had there has been much more emphasis on the fundamentals of the subject in reference as to how to do the examples than there has been on the application of the knowledge acquired. The study conducted in connection

with the material presented in the table showed likewise that there is more need for additional drill procedures in addition and subtraction. The fundamental knowledge required to master the concepts of addition and subtraction is more complex and more difficult for the immature mind to grasp. They differ more widely in needed mastery than does multiplication and division and thus present the pupil with added difficulties. It may likewise be assumed that there is a need for more careful introduction of the fundamentals so as to insure the pupil's retaining that which will be of paramount use to him in subsequent dealings with algebra. The investment of more time and more careful presentation of the fundamentals of the subject will return higher dividends in the end.

It is of interest to note the various types of errors that were made in the tests and the frequency of their occurrence, and the rank of difficulty of each. These data have been tabulated in the following table. The errors in addition and subtraction are tabulated in the first portion and the errors in multiplication in the second section. The errors which were made in the problem and the equation and formula tests have not been tabulated as the types of errors were too difficult to discern and would not lend any instructional material to the study. Those errors which are characterized as "miscellaneous" are those which did not seem to justifiably fall under any of the categories.

Table II.

TYPES OF ERRORS AND FREQUENCY OF OCCURENCE IN
THE HOTZ ALGEBRA SCALES, FORM A

Type of Error	Frequency of Occurrence	Rank of Difficulty
Addition and Subtraction		
Fractions (fundamental errors)	35	1
Removal of parentheses	24	2
Adding unlike terms	20	3
Adding exponents	15	4
Signs	6	5
Cancellation in fractions $\frac{\cancel{x}}{\cancel{x}+1}$	3	6
Not uniting terms	2	7
Leaving letter out of answer	1	8
Miscellaneous	8	-
Multiplication and Division		
Fractions (fundamental errors)	18	1
Cancellation in fractions	16	2
Fractional exponents	14	3.5
Multiplication of binomials	14	3.5
Multiplication errors	10	5.5
Division errors	10	5.5
Signs	9	7
Raising to higher power	6	8
Miscellaneous	6	-

It is well to note in Table II the large amount of errors that

are tabulated under the caption of "fractions". It is difficult to arrive at any conclusions as to the reason or reasons of this fact which stands out so boldly. Multiplication and division seems to lend itself much better to the acquisitive powers of this group than does addition and subtraction. This was later shown to be true in other work that was presented to the group. The errors made were of different types but they all dealt with the fundamentals of the processes of fractions. The large number of errors which were made in the adding of unlike terms shows a decided fallacy in the introduction of algebra to the pupil. In arithmetic he learns that this is an impossible procedure to follow so it would seem that he does not recognize like and unlike terms in algebra as he should. In the pupil's mind there is not the differentiation between the rules of exponents in addition and multiplication that is necessary for correct solving of the examples. The two processes are not differentiated and, it was found in the analysis of the errors, that in the addition of such numbers as 3r, 9r, and 7r that the answer arrived at was fifteen times written as $19r^3$, in short, that the exponents as well as the coefficients were added. The errors made in the cancellation of letters in fractions, as $\frac{\cancel{x}}{\cancel{x}+1}$, were caused by the fact that the pupils were unfamiliar with algebraic fractions at the time when the tests were administered. The same cause is the reason for the large number of errors dealing with fractional exponents. In the multiplication of binomials various errors were found, among them may be noted: multiplication by only one letter in the multiplier, neglecting to add similar terms, multiplying the number-letter combinations but neglecting to multiply by the numbers in both multiplier and multiplicand, neglecting to place the letter with its coefficient, and various miscellaneous errors which were so vague

that it is difficult to characterize them. Multiplication and division errors were noticeably large. This would seem to point toward poor mastery of fundamentals in earlier years of arithmetic where one hundred per cent accuracy should be strived for in order to insure lasting mastery. The difficulty with signs seems about equally divided between the two processes, although in later work it was found that more errors were made in addition and subtraction than in multiplication and division. Raising to a higher power, $(--3xy^3)^4$, although not accounting for many errors was only attempted by six, each one accounting for an error, falls in the category of lack of experience on the part of the pupil for this type of example.

Notes were taken during the teaching review of fundamentals in order to determine the various difficulties which the groups had to contend with in this phase of work. A table has not been made of these errors; rather those which appeared a relatively large number of times were considered as worthy of note for future teaching. It would seem logical to assume that these types of difficulties would be well to consider in the teaching of beginners in algebra. If emphasis were given to these difficulties, in all probability the resulting reactions of pupils would be toward a better mastery and understanding of the subject. The following difficulties may well be noted:

I. Addition.

1. Signs. The facts concerning like and unlike signs cannot be emphasized too strongly at the beginning of the course. The writer found, in reviewing this topic, that the use of mnemonic devices of money or other factors that were within the ken of the pupil was a valuable aid.

2. Adding of like terms. Special emphasis should be placed on

exponents. The pupil will often find it difficult to understand why it is impossible to add squares and cubes. By dividing cubes and squares respectively into a, a, a, and a, a, it will aid the pupil to reason for himself the utter impossibility of doing this. This fallacy was very noticeable.

3. Coefficients. The most frequent error.

4. Carelessness in writing.

5. Copying.

6. Errors in the process of addition.

II. Subtraction.

1. Signs. This formed the greatest number of errors. Mnemonic devices of the thermometer, distance, and the making of change are aids in the mastery of this fundamental. One major reason for the difficulty that pupils find in doing subtraction is in the fact that often their introduction to it in elementary grades is so different that they do not master it as quickly as the teacher believes they will. The reorganization of methods in the earlier years of arithmetic teaching will alleviate much of this, however, and these changes are noticeably coming into use.

2. The removal of parentheses. Care should be taken not to include "nests" of parentheses as there are no algebraic values concerned here that are of absolute necessity.[1]

3. Similar terms: that which is true in addition applies here as well.

4. The other errors which appeared under addition occurred less frequently in subtraction.

III. Multiplication.

---

[1] Report on, Reorganization of Mathematics in Secondary Education (Part I), p. 34.

1. In multiplying by a binomial a large per cent did not multiply by both numbers. This fallacy may be rectified to some degree by introducing the unit through using numbers and progressing from this stage to the use of letters.

2. Adding exponents. Examples of the type $a^n \cdot a^2 = a^{n+2}$ caused a great deal of difficulty, showing that the rule may have been known but that a true conception of the significance of the rule was not understood.

IV. Division.

1. Signs. In both multiplication and division the signs gave the greatest difficulty. The reasons for this are difficult to diagnose as in many aspects it would seem that the rules are easier to notice and learn than in the other fundamentals.

2. Exponents offered difficulties of the same type that occurred in multiplication.

3. The errors were the most difficult to analyze in III and IV than in I and II, probably due to the fact that they combine many of the factors concerned in the latter and therefore are difficult to differentiate.

Some of the above errors were noticed in a study made by M. MacRae and W. L. Uhl[1] and would therefore seem of wide enough scope to be worthy of consideration by those teaching elementary algebra. Also in connection with this study there may be one noted which was conducted by W. A. Coit[2] which showed the similarity of errors in high school with those made in college in the four fundamental processes. This study showed that there was direct correlation between the college and high school thus showing the desirability of early mastery of the fundamentals. It was felt by the writer that there has not been in the past enough emphasis placed on the fundamental principle

---

[1] McRae, M., and Uhl, W. L., "Types of Errors and Remedial Work in the Fundamental Processes of Algebra," Journal of Educational Research, XXVI, (Sept., 1932), pp. 12-21.

[2] Coit, W. A., "Demonds of Elementary Mathematics," School Science and Mathematics," XXIX, (Jan., 1929), pp. 50-58.

of frequent repetition of a unit for mastery.

The majority of the group was deficient in factoring. Typical errors were noted in the various cases so as to determine upon what the emphasis should be placed in future teaching. The teachers in the system were also asked in what phases of the unit they had noticed the largest number of errors and what these errors were. The prevailing errors and difficulties appear here:

I. Monomial factors.

1. Dividing through by the highest possible factor; example, $a^2b^2c+a^3b^4c^3+a^5b^3c^2=a^2b^2c(1+ab^2c^2+a^3bc)$.

In this type of example, often the pupil would divide through by, abc, instead of as written. Likewise, the error was made of not writing the numeral, 1, if the division denoted unity. When coefficients appeared in the example there were but few mistakes noted in this respect.

2. The following type of example caused the more difficulty of the two: $ac+bc+ay+by=(a+b)\ (c+y)$, and, $bd+by-cd-cy=b(d+y)-c(d+y)=(b-c)\ (d+y)$. The difficulty seemed to be in determing what letters should be taken out and also what the remainder should be. The second form offered the added difficulty of remembering to divide by a minus (-) letter.

II. Trinomial square.

1. Difficulty in rearranging terms so as to more easily recognize the perfect square. In general, however, this case did not present the group with many noticeable difficulties.

III. Difference of two squares.

1. Errors were made in the changing of signs within parentheses when preceded by a minus sign as, $(a-b)^2-(d-c)^2=(a-b-d+c)\ (a-b+d-c)$,

which was found to trouble the group less upon the suggestion of inserting an intermediate step which allowed the pupils to more easily see where the signs should be changed; the step: $(a-b)-(d-c)$, $(a-b)+(d-c)$.

IV. Sum and difference of cubes.

1. As this case is not considered an important one by the National Committee, particular note and emphasis was not placed upon it. However, as it was necessary to have the pupils have some knowledge of this case, the members of other classes having been initiated into it, the cubes were studied, but the fifth, seventh, etc., powers were not discussed with the group as a whole. No outstanding errors were noted. In general it may be stated that the procedure itself caused difficulty; the pupils having trouble in remembering the signs and also the second term in the second parenthesis: $x^3+y^3\ (x+y)\ (x^2-\underline{xy}+y^2)$.

V. Trinomials of the Second Degree.

1. This case, according to the National Committee, should be of the type that can be easily factored by trial. Signs offered the most difficulty to the pupils. Two other types of errors offered trouble; (a) finding the correct factors of the end term but when added not giving the second term, and (b) the incorrect factors of the end term which added to give the second term.

A diagram proved valuable as a mnemonical device to many of the pupils in this case. It was chosen only after it was found that many of the pupils made the error recorded under caption, (a).

Example: x^2-x-12, $\overset{6}{-2}\quad \overset{-6}{2}\,\overset{-3}{4}\,\overset{-4}{3}$. The pupils were told to place all of the possible factors of the end term in the above manner and then choose the two which would give the middle. Special emphasis was placed on the signs

and the fact that the larger number must have the sign of the middle term. Likewise it was shown that the end term showed whether or not the signs would be alike, and the middle term determined what the signs would be, if alike, as well as the sign which should precede the larger of two numbers when they were unlike. Increase in ability to do the examples correctly was noticed after the use of this device was started. As this was not conducted as an experiment it is not known whether this was the factor that determined the change or whether past knowledge plus repetition of the exercises accounted for it. It at least was helpful and may, with various modifications depending on the class, be an aid in introducing the unit for the first time.

A program for the remedial treatment of the students in the group was formulated. Review work in the fundamentals was immediately begun so as to insure a basic knowledge for future work. Two methods were used as the groups were of varying sizes. The larger group was divided into smaller groups of pupils who were experiencing the same type of difficulties so as to facilitate ease of teaching and also to allow pupils to aid each other. The smaller group worked at individual paces. However, when it was ascertained that a pupil was capable of going along with more rapidity than the group in which he was situated he was allowed to do so. The program was made elastic enough to allow for changes at any time.

The individuals within each group were allowed to progress at the pace suited to individual talents. The purpose was to salvage as many in the group as possible so as to allow them, through additional work later, to be on a close parity with the other members of the classes from which they originally came. In general, as soon as an individual felt that he was capable of doing a specified phase of work he was given a test on that and the fact that the larger number must have the sign of the middle term. Likewise it was shown that the end term showed whether or not the signs would be alike, and the middle term determined what the signs would be, if alike, as well as the sign which should precede the larger of two numbers when they were unlike. Increase in ability to do the examples correctly was noticed after the use of this device was started. As this was not conducted as an experiment it is not known whether this was the factor that determined the change or whether past knowledge plus repetition of the exercises accounted for it. It at least was helpful and may, with various modifications depending on the class, be an aid in introducing the unit for the first time.

A program for the remedial treatment of the students in the group was formulated. Review work in the fundamentals was immediately begun so as to insure a basic knowledge for future work. Two methods were used as the groups were of varying sizes. The larger group was divided into smaller groups of pupils who were experiencing the same type of difficulties so as to facilitate ease of teaching and also to allow pupils to aid each other. The smaller group worked at individual paces. However, when it was ascertained that a pupil was capable of going along with more rapidity than the group in which he was situated he was allowed to do so. The program was made elastic enough to allow for changes at any time.

The individuals within each group were allowed to progress at the pace suited to individual talents. The purpose was to salvage as many in the group as possible so as to allow them, through additional work later, to be on a close parity with the other members of the classes from which they originally came. In general, as soon as an individual felt that he was capable of doing a specified phase of work he was given a test on that portion and, if satisfactorily passed, he continued his work. The incentive to regain lost prestige and be on a par with others in the group caused many of them to work hard. Each one understood that he could go on to the next portion of work as soon as he had mastered that which he was doing. Although there were some who attacked the plan with little or no enthusiasm, soon the majority of them saw that it depended upon himself individually as to how rapidly he would go ahead. Conducting a class in this manner takes a great deal of the teacher's time and much repetition of material, but it seemed to have produced reasonably good results. It was deemed wise to have various types of check-ups each week so as to insure that there was not any undue shirking on the part of those who did not have the incentive to progress at a very rapid pace.

At the outset one outstanding difficulty was noticed: the text did not lend itself to the group as efficiently as it should. Questions regarding fundamental rules of algebra were asked and it was found that the pupils did not have a knowledge sufficient enough to cope with the work that they would soon find themselves doing. Upon studying reasons for this the writer found that the text did not contain the necessary rules. The pupils had been requested to take notes in class concerning rules that they would need but through carelessness, absence, and misunderstanding they did not have many of them. Neither did the text contain any preliminary exercises when the pupils were initiated into a new unit. It is difficult for pupils of this age to keep adequate notebooks unless they are continually checked by the teacher and this necessitates a great deal of work and time that could be used to a better advantage in other ways. It would seem, therefore, that a more adequate text containing rules and preliminary exercises would have done

much to eliminate some of the errors that this group had been making.

In connection with this factor of the pupil's not having the necessary work in their notebooks or text the problem of absences may well be mentioned. As work progressed it was noticed that many of the pupils, upon being asked why certain phases of the work in algebra presented difficulties, said that they were absent at the time that the unit or topic had been discussed in class. The writer decided to ascertain the number of times that the pupils had been absent during the year. The office records were consulted for this purpose with the following results:

1. The absences ranged from 3 to 23 days.
2. The average was 11.47 days for the period.
3. Tardiness was not considered, but as the range was from 0 to 10 and this meant that those tardy were often absent from the first period when their algebra was conducted, it may have played a negative part toward the accumulation of needed knowledge.

It is impossible to draw any positive conclusions as there are so many irrelevant factors that may have entered into the fact that there was such a decided number of absences among a failing group. However, it is of interest to notice a few factors regarding the absences. Although the administration provided a system for checking on the make-up of absentees, for some reason all of the pupils either did not avail themselves of the opportunity or were not given enough individual aid. The failing pupil needs more individual instruction than does the brighter pupil.[1] It does not seem unreasonable to expect that through proper help these pupils, even though absent a number of times, would not be able to attain and retain more of the subject-matter than they did. The absent pupil is at a distinct

---

[1] Miller, Joseph, "Causes of Failure and Success in School," Educational Method, X, (March, 1931), p. 329.

disadvantage and the teacher who realizes this will make provisions for the pupil. The utter lack of knowledge concerning some of the important facts seemed to point toward a lack of individual instruction which is quite necessary. There may also have been a lack of cooperation on the part of the pupil, in that he did not desire to do the make-up work with the proper spirit. Often an absence is considered a "day off" and there is no desire on the part of the pupil to regain the lost ground. The subject of absences and make-up work is an important phase of teaching and cannot well be slighted. There is an opportunity in this field for much constructive work to be done by teachers and administrators. A study of the various causes for failures among pupils is not complete without an attempt to view the problem of failing from the standpoint of the pupil himself. Individual psychology plays a very important role in failure as it does in success. An attempt was made to determine other phases than intelligence in the pupil that went toward causing him to be a failure. Among these personal difficulties and assets were noted initiative, accuracy, industry, cooperativeness, perseverance, efficiency, poise, adaptability, neatness, responsibility, and promptness. These personal traits enter into success or failure of a pupil depending upon the degree to which he does or does not possess them, respectively. These will be noted more specifically in the discussion of a few case studies which were made. The general definition of each is as follows:

1. Initiative. The pupil with initiative responds without being urged. He adds to the class recitation and notes connection between work discussed and its application to life situations; as in positive and negative numbers. The opposite extreme is the routine worker who reads

passages from the text and listens to the class discussions but seldom offers contributions to the group voluntarily. His work is often well done but he neglects to see the applicability of the material. He is dependent upon the teacher when doing new work as he does not digest fully that which is in the text. Evidence of the presence or absence of this trait can readily be inferred from the effort made by the individual pupil to explain life-situations which are discussed in connection with various phases of the work.

2. Accuracy. "The correlation between accuracy and success in the unit of positive and negative numbers is almost perfect."[1] In dealing with algebra in general, there is a decided need for accuracy on the part of the pupil to assure success in doing examples and problems. Carelessness in copying, too hurried reading of problems, and too rapid doing of exercises lead to inaccuracy. These failings were all noticed in various members of the group. Insistence on checking results received tend to overcome many of these errors of inaccuracy but there must be a desire on the part of the pupil to correct these in order to insure satisfactory results.

3. Industry. The industrious pupil is busy throughout the entire period. He is interested in getting the work finished and, at the same time, doing a good piece of work. He does not have to be told what to do if all the work has not been assigned but rather does he find more of the same type to do and keeps himself busy. The lazy pupil has to be continually spoken to about not working and wasting time. Industry begets work accomplished which is a decided asset to the pupil.

4. Cooperativeness. Orders and directions are followed willingly by the cooperative pupil. The management of the class is made much easier

---

[1] Waples, D. and Stone, C., The Teaching Unit, p. 127.

by this type of pupil. Routine assignments and directions are often done in as many different ways as there are pupils when this trait is found to be lacking. The teacher is continually harassed by the noncooperative pupil because of the necessity of continued repeating of directions which would not be necessary if better attention were given by the pupil. This trait should be developed by the individual as it will be a requisite in life work.

5. Perseverance. The persevering pupil better adapts himself to the work than does the impatient one. He realizes the necessity of reading material pertaining to that which he is to do and is therefore able to see how to apply the knowledge that he derives from the printed page. The impatient pupil will hurriedly skip over the explanations that come at the beginning of a new type of exercise and, upon plunging into the exercises or problems, finds that he is unable to begin and immediately finds it necessary to depend upon the teacher. Such hurried departures into parts unknown is a very doubtful procedure and is detrimental to the pupil's growth. Through correct perseverance the pupil will be able to add a great deal to his success in school and life.

6. Efficiency and Poise. Efficiency is the ability to so arrange work as to get the maximum results with a minimum expenditure of effort. The pupil uses short-cut methods and maintains a more effective mastery of the subject than does the inefficient one. Poise gives the pupil self-confidence and he has a deliberate attitude toward getting the work done.

7. Adaptability. This trait is comparable to cooperativeness in that the pupil easily and willingly adapts himself to new situations and is a teachable type of person. The opposite type interferes with the class

progress by being too argumentative and defending inefficient methods of procedure when better methods have been devised by the teacher.

8. Neatness. Neatness in itself is an aid but not a necessity. Often too much emphasis is put on neatness and causes a feeling of dissension among the pupils as they feel that partiality is being shown. However, neatness is essential insofar as accuracy is concerned. It may be noted that many errors are made because a pupil is unable to read what he has copied or, in rapid and not careful erasures, he reads the wrong numbers.

9. Responsibility. The responsible pupil is both cooperative and efficient. He does not need continual watching by the teacher. He shows more intelligence concerning assignments and class work than does the irresponsible one. This trait is of paramount importance throughout life. He is much more apt to be accurate as he sees the necessity of checking the results that he obtains in his work.

10. Promptness. The prompt pupil is ready to begin work at the beginning of the class period. He has the proper materials ready for immediate use and does not waste unnecessary time. He is not dilatory in passing in assignments and thus he is able to give full attention to the work at hand. The dilatory pupil often finds that he is so far behind in his work that it is either impossible for him to reach the standard of the class or else much of the work has to be done in a mediocre fashion. The study habit of the prompt pupil is likely to be advanced and he realizes what he should do and appoints special hours in which to do his work.

In connection with the above discussion dealing with the various traits of individuals, it would not be amiss at this time to note two cases of pupils in the groups that were studied and the prevalency or absence of

the traits. All the cases were noted in the same manner in which these two will be discussed, but these two are more interesting and show the possibilities of increasing the mental output of a pupil through individual treatment and a show of interest in the pupil. A pupil who receives personal interest from the teacher will generally reciprocate with a growing interest in the teacher and consequently an interest in the subject, at least to the degree of endeavoring to do a better grade of work.

CASE I

This pupil has an average I. Q. and is able to do average work in algebra. At the outset he made it apparent that he thought he was a hopeless person and that it would be of no avail for him to attempt trying to succeed in algebra. An attempt to understand his attitude through conversation received only rebuff in desultory sentences, such as, "What is the use of my doing this? I can't understand it anyway." It was apparent that he had no interest in the subject nor in attempting to pass it. His work was accomplished in spasmodic form. During supervised study he would work with reasonable energy for a limited time and then apparently lose interest, stop working, and gaze about the room. When he was successful in a given procedure he worked rapidly and accurately, but when unsuccessful immediately lost interest, which is a natural psychological result, and had not the initiative nor perseverance to overcome this attitude. The case seemed to be impregnable for two or three weeks. By means of conferences and aiding him to understand and master difficult portions of his work the writer endeavored to change the attitude of the pupil and to show him that, through concentrated effort on his part, he could do good work and gain something from the course. By not being too solicitous to him, but, at the same time,

showing a willingness to aid when he so desired the pupil became aware that the writer was willing to help him providing he would endeavor to reciprocate by helping himself and changing his attitude. There was suddenly a noticeable change in the pupil and his work showed evidence of improvement. His assignments were finished more punctually, he raised questions as to how to do the more advanced work, and he showed a desire to progress in the subject. As tests were given individually when the pupil felt that he was able to cope with them, it was noticed that he made much more rapid progress than he had previously and was on a par with the first of the group. By means of the program of individual progress he was able to finish factoring and be initiated into fractions. His previous difficulty had been in not having certain phases of the work made clear to him and, not being able to do some of the work, he had soon lost interest. Personal aid and an endeavor to understand his individual case had remedied his failing and he was able to surpass the average work of the pupils in his group. Each individual is so radically different that there must be an understanding of this fact by the teacher and improved methods of psychology and teaching techniques must be used to combat the element of failure among the pupils. It is not enough to present material so that the majority can understand the work. It must be so presented that all those who have any native intelligence can understand the work as presented. Either a system should be evolved wherein individual attention can be given or else there should be provided a system of ability grouping in the subject so that the pupils will be on more of a parity than is true of the average heterogeneous group. This idea seemed more and more imperative as the writer dealt with the groups.

CASE II

This pupil likewise has an average I. Q. and is capable of doing average or above average work in algebra. He is a relatively slow thinker and his muscular-mental coordination is not well developed. Different from the above case he was interested in acquiring a knowledge of algebra and had perseverance, responsibility, and a fair amount of initiative. The group with which he had been studying had been progressing too rapidly for him and, therefore, as time went on he was losing more and more of the necessary fundamentals that were necessary for him to master to a degree. He was willing to expend effort and time in an attempt to gain a knowledge of algebra. Observations of the class showed that the pupils were being given too advanced work and that the teacher was too demanding of them. The class was far in advance of the other classes in the building and this rapid advancement had caused them to be unable to assimilate properly the fundamental concepts of the subject. They were introduced into new material before adequate time for drill for mastery had been presented and, therefore, as they progressed they were not sure of the ground that had been covered. The classes were conducted too much on the pattern of college classes for the mentality of the high school pupil to cope with to any degree of success. These factors had caused, in part, the necessity of his receiving special attention. Through concentrative work on that which caused him the most difficulty it soon became apparent that he was ready for a test on the material covered. It was here that the most noticeable difficulty lay; he lacked confidence in his ability. Regardless of how well he seemed to have mastered the material before taking an examination, upon working examples under pressure of time he would fail disastrously. There were two factors

that had to be improved before it could be hoped that he could cope with algebra: self-confidence and ability or skill to do his work more rapidly. The latter failing was considered first as it was hoped that through the acquisition of the skill that he would gain in self-confidence. Intensive drill material in regular drill units were administered and improvement in speed of computation was noticed. During private interviews the pupil was encouraged and through these discussions his confidence was increased to a marked degree. The first departmental test which was returned he received a grade of 4 out of a possible 25. The grades increased each successive week until it had reached the high point of 20 which shows a decided improvement for the time spent in individual work with the pupil. Elation was felt both by the writer and the pupil. Although considerable time was spent at the outset, soon the pupil needed only occasional aid as confidence in his ability improved. He likewise gained a great deal more from his classes which was shown in the questions he asked and the quickness with which he grasped new ideas. Aid was also given in short-cut methods of computation of which he seemed to have had but little training. Some of his difficulty can be traced to lack of proper training in arithmetic and much of the rest to too rapid advancement into new fields that he was unable to grasp. This attempt at redeeming a pupil was well worth the work expended, in the writer's mind, although, it may be added, it would have been much more expedient to have made provisions for a similar type of work earlier in the pupil's career and to have made provisions for a remedial period within class hours before the pupil was so far behind some of the group.

It is not meant to infer that the two cases above are character-

istic of all the pupils in the class. Some made reasonably good progress in the time that they were in the special groups while there were others who made little or no progress. It must be realized that some pupils have not the faculty to master algebra to any marked degree. In some of these it is a lack of sufficient native intelligence, perseverance, initiative or other traits that were discussed previously. It would seem wise to make provisions in some manner through administrative means to take pupils who are noticeably unable to cope with the subject into conference and direct them into fields wherein they can be rewarded with a fair amount of success. They are not only wasting their own time and that of the teacher, but are also building up within themselves a complex about failure and inferiority that will prove a detriment to them in years to come.

Boston University
School of Education
Library

GENERAL CONCLUSIONS

In order to achieve the best results in the teaching of algebra, some form of individual instruction is necessary. The individual differences within a group must be coped with in some systematic order so that each pupil will receive the greatest amount of desirable knowledge from the course. The proven values of regular classroom recitations should be retained, however, but there is nothing in an individual plan which prevents this. Within the class period the teacher should think in terms of the individual as any drill subject needs individual instruction.

The important function involves a study of child psychology and the application of this knowledge to the individual. Whatever system reveals most about the child best subserves this end. The individual plan without a doubt does present the greatest opportunity for the acquirement of this knowledge. Individual teaching is not new; good teachers have always made use of it.

The securing of better scholarship is, after all, a matter of teaching efficiency. The success of any method is therefore dependent upon its teaching staff. Individual differences still persist in any method and must be met and studied. The individualizing of instruction to the extent of individual and graded assignment so that each pupil should perform in accordance to his capacity should be the ideal of any method of instruction.

CONCLUSIONS ON STUDY

1. Individual instruction, applying material to the individual in such a manner that he will grasp the work, is an important phase of instruction in algebra.

2. Attempts are made to teach too much material to pupils in algebra.

3. The failing students had far more difficulty with the material that was suggested to be pruned from the study of algebra. It was likewise noted that forty-one per cent of the pupils failed in that which could well have been omitted from the course of study and that a large portion of the others would have been in the passing group had the time that was spent on the unessentials been spent on that which is essential in the course.

4. The salvaging of all pupils in a course is probably impossible, but there need not be the terrible toll on pupils taking algebra if more consideration is given to individual instruction.

5. Absences, or not the correct type of make-up work, faulty texts, and personal traits of the pupil play a large part in subsequent failures.

FIRST YEAR ALGEBRA SCALES

SERIES A

BY HENRY G. HOTZ

Published by Teachers College, Columbia University. Copyright, 1920, by Teachers College

---

School.................... Date.................... City.................... County....................

Write your name here.. Age....................

When did you begin to study algebra? Month.. Year....................

ADDITION AND SUBTRACTION

Carefully perform the operations as indicated.

1. $4r + 3r + 2r =$

3. $12b + 6b - 3b =$

5. $7x - x + 6 - 4 =$

9. $(4r - 5t) + (s - 3r) =$

10. $8c - (-6 + 3c) =$

12. $5x - [4x - (3x - 1)] =$

13. $\frac{3c}{4} - \frac{3c}{8} =$

15. $\frac{1}{a - x} - \frac{3x}{a^2 - x^2} =$

18. $3 - \frac{3 - 2x}{4} - 2x =$

22. $\frac{3 - 2x}{(x - 1)^3} + \frac{x + 1}{(x - 1)^2} - \frac{1}{(x - 1)} =$

23. $\sqrt{20} + \sqrt{45} + \sqrt{1/5} =$

24. $\frac{a}{a - 2} - \frac{a - 2}{a + 2} + \frac{3}{4 - a^2} =$

50 L-8-29

FIRST YEAR ALGEBRA SCALES

SERIES A

By HENRY G. HOTZ

Published by Teachers College, Columbia University. Copyright, 1920, by Teachers College

---

School.................................... Date.. City.. County..

Write your name here... Age..

When did you begin to study algebra? Month.. Year..

MULTIPLICATION AND DIVISION

Carefully perform the operations as indicated. Reduce all answers to their *simplest* forms.

1. $3 \cdot 7y =$

2. $\dfrac{12n}{4} =$

3. $2a \cdot 4ab^2 =$

7. $4x \cdot (-3xy^3) =$

9. $\dfrac{18m^2n - 27mn^2}{9mn} =$

11. $(2a^2 + 7a - 9)\,(5a - 1) =$

16. $(-3xy^3)^4 =$

17. $\dfrac{c^4 - d^4}{(c-d)^2} \cdot \dfrac{c - d}{c^2 + d^2} =$

20. $\dfrac{p^2 + 4p - 45}{p^2 + 2p + 4} \cdot \dfrac{p^3 - 8}{p^2 - 81} \cdot \dfrac{1}{3pr - 15r} =$

21. $\dfrac{x^3 + 27}{x^2 + x - 12} \div \dfrac{3x + 9}{x + 4} =$

22. $64^{\frac{2}{3}} \times 27^{\frac{1}{3}} =$

23. $\dfrac{3\sqrt{6a}}{2a\sqrt{18}} \cdot \sqrt{12a} =$

FIRST YEAR ALGEBRA SCALES

SERIES A

BY HENRY G. HOTZ

Published by Teachers College, Columbia University. Copyright, 1920, by Teachers College

---

School.................... Date.................................. City.................... County......................

Write your name here .... Age......................

When did you begin to study algebra? Month.................................. Year......................

PROBLEMS

Do not work out the answer to the problem—merely *indicate* the answer or *state* the equation in each case.

1. If one coat cost x dollars, how much will 3 coats cost?

Answer...

2. A man is m years old; how old was he r years ago?

Answer...

4. A gold watch is worth ten times as much as a silver watch, and both together are worth $132. How much is each worth?

Equation...

5. The distance from Chicago to New York by rail is 980 miles. If a train runs v miles an hour, what is the time required for the run?

Answer...

7. The total number of circus tickets sold was 836. The number of tickets sold to adults was 136 less than twice the number of children's tickets. How many were sold of each?

Equation...

8. A rectangular box is d inches deep, w inches wide, and contains r cubic inches. What is its length?

Answer...

9. The area of a square is equal to that of a rectangle. The base of the rectangle is 12 feet longer and its altitude 4 feet shorter than the side of the square. Find the dimensions of both figures.

Equation...

(Turn over. There are problems on the other side)

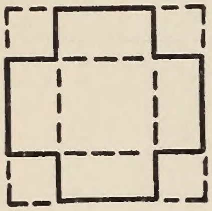
12. A train leaves a station and travels at the rate of 40 miles an hour. Two hours later a second train leaves the same station and travels in the same direction at the rate of 55 miles an hour. Where will the second train pass the first?

Equation..

13. A merchant has two kinds of tea, one kind costing 50 cents and the other 65 cents per pound. How many pounds of each must be mixed together to produce a mixture of 20 pounds that shall cost 60 cents per pound?

Equation..

14. An open box is made from a square piece of tin by cutting out a 5-inch square from each corner and turning up the sides. How large is the original square, if the box contains 180 cubic inches?



Equation..

FIRST YEAR ALGEBRA SCALES

SERIES A

BY HENRY G. HOTZ

Published by Teachers College, Columbia University. Copyright, 1920, by Teachers College

---

School.................... Date.................... City.................... County....................
Write your name here.. Age....................
When did you begin to study algebra? Month.. Year....................

EQUATION AND FORMULA

Solve the following equations and formulae:

1. $2x = 4.$

2. $7m = 3m + 12.$

4. $5a + 5 = 61 - 3a.$

6. $10 - 11z = 4 - 8z.$

8. $c - 2\,(3 - 4c) = 12.$

11. The area of a triangle $= \frac{1}{2}bh$, in which
$b =$ length of the base
and $h =$ height of the triangle.

How many square feet are there in the area of a triangle whose base is 10 feet, and whose height is 8 feet?

14. $3m + 7n = 34$
$7m + 8n = 46$

18. $\dfrac{x + 3}{x - 2} = \dfrac{x + 5}{x - 4}.$

19. $p^2 - 5p = 50.$

23. $\dfrac{6x - 2}{x + 3} - 3 = \dfrac{3x^2 + 13}{x^2 - 9}.$

24. $S = \frac{1}{2}gt^2$; solve for t.

25. $\sqrt{x^2 - 1} - x = -1.$

BIBLIOGRAPHY

Barber, Harry C., Reorganized Mathematics in the Junior High School, Second Yearbook, National Council of Teachers of Mathematics, pp. 120-130.

Beatley, Ralph, Effect of Varying Amounts of Arithmetic, Geometry, and Algebra Studied in the Junior High School, Second Yearbook, National Council of Teachers of Mathematics.

Benz, H. E., Summary of Some Scientific Investigations of Teaching of High School Mathematics, Eighth Yearbook, National Council of Teachers of Mathematics, pp. 14-46.

Billett, R. O., What The High Schools Are Doing for the Individual: Preliminary Findings of National Survey of Education, School Life, XVI, January, 1931, pp. 85-87.

Chateauneuf, A. O., Changes in the Content of Elementary Algebra Since the Beginning of the High School Movement as Revealed by the Textbooks of the Period, National Survey of Education Bulletin, 1932, No. 17, Monograph No. 13, pp. 140-142.

Coit, W. A., Demons of Elementary Mathematics, School Science and Mathematics, XXIX, January, 1929, pp. 50-58.

Dalman, M. A., Hurdles, A Series of Calibrated Objective Tests in First Year Algebra, Journal of Educational Research, I, January, 1920, pp. 47-62.

Douglass, H. R., Modern Method in High School Teaching, Houghton Mifflin Company, Boston, 1926, xviii, 554 pp.

Edwards, W. H., Need for Testing Practices in Curriculum Readjustment, Mathematics Teacher, XXII, October, 1929, pp. 320-322.

Gadske, R. E., Comparison of Two Methods of Teaching First Year Algebra, School Science and Mathematics, XXXIII, June, 1933, pp. 635-640.

Gates, A. I., Psychology for Students of Education, Macmillan Company, New York, 1923, xvi, 489 pp.

Hirsch, E. C., Case Method of Dealing with Individual Difficulties in the Secondary School, School Review, XXXVIII, September, 1930, pp. 525-531.

Holy, T. C., and Sutton, D. H., Ability Grouping in Ninth Grade, Educational Research Bulletin, IX, October 22, 1930, pp. 419-422.

Irwin, M. E., Caring for Individual Differences, Educational Research Bulletin, IX, December, 1929, pp. 10-20.

BIBLIOGRAPHY (cont.)

Jensen, M. D., and Jensen, D. W., The Influence of Class Size Upon Pupil Accomplishment in High School Algebra, Journal of Educational Research, XXI, February, 1930, pp. 120-137.

Johnson, J. T., Adapting Instructional Material to Individual Differences in Learning, Mathematics Teacher, XXVI, April, 1933, pp. 193-199.

Judd, C. H., Psychological Explanation of Failures in High School, Proceedings of High School Conference, Illinois University Bulletin, 1931, pp. 222-226.

Lide, Edwin S., Instruction in Mathematics, National Survey of Secondary Education Bulletin, 1932, No. 17, Monograph No. 23, vi, 72 pp.

MacRae, M., and Uhl, W. L., Types of Errors and Remedial Work in the Fundamental Processes of Algebra, Journal of Educational Research, XXVI, September, 1932, pp. 12-21.

Miller, Joseph, Causes of Failure and Success in School, Educational Method, X, March, 1931, pp. 327-333.

Morrison, H. C., The Adaptation of Instruction to the Needs, Interests, and Capacities of Students, Thirteenth Yearbook, National Society for the Study of Education, pp. 9-31.

Murphy, C. D., Problems of Algebra Pupils, Proceedings of High School Conference, Illinois University Bulletin, 1930, pp. 209-214.

New Hampshire State Board of Education, Program of Studies.

Pugsley, C. A., Reducing and Handling School Failures, American School Board Journal, LXXXVI, March, 1933, pp. 18-20.

Reorganization of Mathematics in Secondary Education, Houghton Mifflin Company, Boston, 1927, xi, 181 pp.

Ryan, H. H., Adapting Instruction to Individual Differences, Junior-Senior Clearing House, VII, September, 1930, pp. 34-38.

Schorling, Raleigh, A Tentative List of Objectives in the Teaching of Junior High School Mathematics with Investigations for the Determination of Their Validity, George Wahr, Ann Arbor, Michigan, 1925, 137 pp.

Schorling, Raleigh, Finished Report of the Committee on Individual Differences, Mathematics Teacher, XXV, November, 1932, pp. 420-426.

Schultze, Arthur, The Teaching of Mathematics in Secondary Schools, Macmillan Company, Boston, 1912, xx, 370 pp.

BIBLIOGRAPHY (cont.)

Shannon, J. R., Three Dimensions in Curricular Adaptations, Education, LI, December, 1930, pp. 193-198.

Sutherland, A. A., Factors Causing Maladjustment of School to Individual, Twenty-fourth Yearbook, National Society for Study of Education, p. 3-90.

Sykes, M., Study of Individual Differences as They Effect Mathematics Classes, Mathematics Teacher, XXIV, March, 1934, pp. 180-185.

Symonds, P. M., Measurement in Secondary Education, Macmillan Company, New York, 1927, xvii, 588 pp.

Thorndike, E. L., Psychology of Algebra, Macmillan Company, New York, 1926, xi, 483 pp.

Waples, D., and Stone, C., The Teaching Unit, D. Appleton and Company, New York, 1929.

Wetzell, W. A., Teaching Technique and the Size of the Class, School Life, XV, June, 1930, pp. 181-182.

BOSTON UNIVERSITY
1 1719 02546 0413